SPLAYN-FUNKSIYALARINING YAQINLASHISH KO‘RSATGICHI BO‘YICHA TAQQOSLASH

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Annotatsiya

Dunyodagi ko‘plab ilmiy tadqiqotlarda, raqamli signallar bilan ishlashda splayn funksiyalaridan foydalanish keng qo‘llaniladi. Hozirgi vaqtda raqamli signallarni tanlashning ko‘plab usullari mavjud. Ularga misol qilib Lagranj va Nyuton interpolyatsion usullari, splayn funksiya, Fure qator va ko‘rsatkichli funksiyalarni yaqinlashtirish usullari, chiziqli tenglamalar tizimini yechishning ortogonal usullari, diskret to‘lqinli transformatsiyalar va bir qator boshqa interpolyatsion usullari kiradi. Tadqiqot davomida teng oraliqlar uchun kubik B-splayn funksiyasi va lokal Natural splayn-funksiyasi tanlab olingan. Ushbu maqolada funksiyalarni interpolyatsiya jarayoni lokal kubik splayn funksiyalarni berilgan analitik funksiyalarga yaqinlashishi batafsil yoritilgan.

Tayanch so‘zlar: Lokal splayn-funksiyalar, lokal Natural splayn-funksiya, interpolyatsiya jarayoni, kubik B-splayn, analitik funksiyalar.

СРАВНЕНИЕ СПЛАЙН-ФУНКЦИЙ ПО ИНДЕКСУ ПРИБЛИЖЕНИЯ

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Аннотация

Во многих мировых научных исследованиях широко применяется использование сплайн-функций при работе с цифровыми сигналами. В настоящее время существует множество способов выбора цифровых сигналов. К ним относятся методы интерполяции Лагранжа и Ньютона, методы аппроксимации сплайн-функцией, рядами Фурье и показательной функцией, ортогональные методы решения систем линейных уравнений, дискретные вейвлет-преобразования и ряд других методов интерполяции. В ходе

исследования кубическая В-сплайн-функция и локальная естественная сплайн-функция были выбраны для равных интервалов. В этой статье подробно описывается процесс интерполяции функций, который аппроксимирует функции локального кубического сплайна к заданным аналитическим функциям.

Ключевые слова: Локальные сплайн-функции, локальная естественная сплайн-функция, интерполяционный процесс, кубический В-сплайн, аналитические функции.

Dunyoda tez rivojlanayotgan axborot-kommunikatsion texnologiyalarida aniqlangan signallarni tiklash, raqamli ishlov berish algoritmlaridan foydalanish va optimal yechimlarni topish uchun yuqori unumli, samarali hisoblash usullarini ishlab chiqish muhim ahamiyat kasb etmoqda. Jumladan lokal splayn-funksiyalarni qurishda hisoblar sonining kamligi, yuqori aniqlik darajasi, raqamli ishlov berish algoritmlarining moslashuvchanligi, optimal differensial va ekstremal xossalari, parametrlarni hisoblashning soddaligi tufayli signallarni raqamli ishlash algoritmlarini yaratishda muhim matematik vosita hisoblanadi [1-3], [6,7]. Quyida bir nechta lokal splayn funksiyalarini qurilishini keltirib o‘tamiz.

n – darajali ixtiyoriy $S_n(x)$ interpolyatsiyalanadigan $f(x)$ funksiya quyidagi ko‘rinishga ega bo‘lgan B-splaynning [a:b] oraliqdagi formulasini keltiramiz [19-21]:

$$f(x) \cong S_n(x) = \sum_{i=-1}^{n-1} b_i B_{i,n}(x), \qquad a \le x \le b, \tag{1}$$

Bu yerda *n* – splayn darajasi, b_i – koeffitsiyentlar, $B_{i,n}(x)$ asosiy funksiyani ifodalaydi. Yuqoridagi formula asosida *n=3* holatida kubik B-splayn funksiyasini ko‘rib chiqamiz (2).

$$S_3(x) = b_{-1}B_{-1}(x) + b_0 B_0(x) + b_1 B_1(x) + b_2 B_2(x), \qquad (2),$$

(2) formuladagi $B_{i,n}(x)$ asosiy funksiyaning qiymatlarini quyidagicha hisoblaymiz (3), (4):

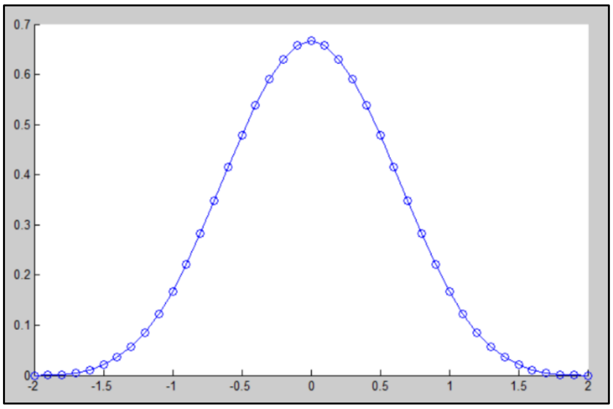
$$B_{i,0}(x) = \begin{cases} 1, & \text{agar } x_i \le x < x_{i+1}, \\ 0, & boshqa\ holatda. \end{cases} \tag{3}$$

$$B_{i,n}(x) = \frac{x - x_i}{x_{i+n} - x_i} B_{i,n-1}(x) + + \frac{x_{i+n} - x}{x_{i+n} - x_{i+1}} B_{i+1,n-1}(x), \qquad (4),$$

$B_{i,3}(x)$ asosiy funksiyani $x \in [0:2]$ oraliqda hisoblaymiz. Bir qancha hisoblashlar va soddalashtirishlardan so‘ng kubik B-splaynning asosiy funksiyasi quyidagi ko‘rinishga ega bo‘ladi (5).

$$B_3(x) = \begin{cases} 0, & x \geq 2, \\ \frac{(2-x)^3}{6}, & 1 \leq x < 2, \\ \frac{1+3(1-x)+3(1-x)^2-3x^3}{6}, & 0 \leq x < 1, \\ B(-x), & 0 < x \end{cases} \qquad (5)$$

Kubik B-splaynning asosiy funksiyasini [-2:2] oralig‘idagi grafigi quyidagi ko‘rinishda bo‘ladi (1-rasm).



1-rasm. Kubik B-splayn funksiyasini [-2:2] dagi grafigi

$S_3(x)$ ni hisoblash jarayonida B-splaynning [0:1] oraliqdagi barcha qiymatlaridan foydalanib, signallarni tiklashga erishiladi.

(1) formuladagi b_i koeffitsiyentlarni hisoblash jarayonida berilgan qiymatlarning qanchadan olinishiga ko‘ra turlicha hisoblanadi [5,6]. Biz berilgan qiymatlar yordamida b_i koeffitsiyentlarni *uch*, *besh*, *yetti* nuqtali formula yordamida aniqlaymiz (6), (7), (8).

b_i koeffitsiyentlarni hisoblashning 3 – nuqtali formulasi:

$$b_i = \left(\frac{1}{6}\right)(-f_{i-1} + 8f_i - f_{i+1}), \qquad (6),$$

b_i koeffitsiyentlarni hisoblashning 5 – nuqtali formulasi:

$$b_i = \left(\frac{1}{36}\right)(f_{i-2} - 10f_{i-1} + 54f_i - 10f_{i+1} + f_{i+2}), \qquad (7),$$

b_i koeffitsiyentlarni hisoblashning 7 – nuqtali formulasi:

$$b_i = \left(\frac{1}{216}\right)(-f_{i-3} + 12f_{i-2} - 75f_{i-1} + 344f_i - 75f_{i+1} + 12f_{i+2} - f_{i+3}), \quad (8).$$

bu yerda, f_i – dastlabki berilgan nuqtalar qiymati.

Analitik funksiyalarni tiklashda natural splayn-funksiyasidan keng foydalaniladi. Quyida uning qurilish usuli keltirilgan [12, 20].

Har bir $[x_i, x_{i+1}](i = \overline{1, n-1})$ oraliqdagi $S(x) = S_i(x)$ funksiyani uchunchi darajali ko‘phad ko‘rinishida qidiramiz (9).

$$S_i(x) = a_i + b_i(x - x_i) + \frac{c_i}{2}(x - x_i)^2 + \frac{d_i}{6}(x - x_i)^3, \tag{9}$$

$$x_{i-1} \le x \le x_i, i = 1,2, \dots, N,$$

bu yerda a_i, b_i, c_i, d_i aniqlanishi kerak bo‘lgan koeffitsiyentlar. Keltirilgan koeffitsiyentlarni aniqlash quyidagicha amalga oshiriladi.

$$S_i'(x) = b_i + c_i(x - x_i) + \frac{d_i}{2}(x - x_i)^2,$$

$$S_i''(x) = c_i + d_i(x - x_i), S_i'''(x) = d_i,$$

yuqoriga asoslanib koeffitsiyentlarni quyidagicha yozib olamiz

$$a_i = S_i(x_i), b_i = S_i'(x_i), c_i = S_i''(x_i), d_i = S_i'''(x).$$

Biz interpolyatsiya shartiga $S(x_i) = f(x_i), i = 1,2, \dots, N$, ko‘ra,

$$a_i = f(x_i), i = 1,2, \dots, N.$$

koeffitsiyentlarni aniqlaymiz $a_0 = f(x_0)$.

Bundan tashqari, funksiyaning uzliksizlik shartini quyidagicha yozib olamiz.

$$S_i(x_i) = S_{i+1}(x_i), i = 1,2, \dots, N-1 .$$

Demak, $S(x)$ kubik splayn uchun ifodalarni hisobga olib, $i = 1,2, \dots, N-1$ ta tenglamalar quyidagicha ko‘rinishda ifodalanadi

$$a_i = a_{i+1} + b_{i+1}(x_i - x_{i+1}) + \frac{c_{i+1}}{2}(x_i - x_{i+1})^2 + \frac{d_{i+1}}{6}(x_i - x_{i+1})^3.$$

Agar $h_i = x_i - x_{i-1}$ belgilash kiritsak, umumiy holatda tenglamalar (10) ko‘rinishi keladi.

$$h_i b_i - \frac{h_i^2}{2} c_i + \frac{h_i^3}{6} d_i = f_i - f_{i-1}, \rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\ i = 1,2, \dots, N \tag{10}$$

endi birinchi tartibli hosila uchun uzluksizlik shartini yozamiz

$$S_i'(x_i) = S'_{i+1}(x_i), i = 1,2, \dots, N-1$$

yuqoridagi almashtirishlardan so‘ng tenglamalar (11) ko‘rinishga keladi

$$h_i c_i - \frac{d_i^2}{2} h_i = b_i - b_{i-1}, \rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\ i = 2,3, \dots, N. \tag{11}$$

Ikkinchi tartibli hosilaning uzluksizlik shartidan (12) tenglamalarni olamiz

$$h_i d_i = c_i - c_{i-1}, \rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\ i = 2,3, \dots, N. \tag{12}$$

(10) - (12) larni birlashtirib, 3N noma’lum $b_i, c_i, d_i,\ \ i = 1,2, \dots, N$ uchun 3N - 2 tenglamalar sistemasiga ega bo‘lamiz.

Ikkita yoqolgan tenglamani $S(x)$ kubik splayn uchun u yoki bu chegara shartini o‘rnatish yo‘li bilan olish mumkin. Masalan, $f(x)$ funksiya $f''(a) = f''(b) = 0$ shartini qanoatlantiradi deylik. Keyin tabiiy ravishda aniq $S''(a) = S''(b) = 0,$ bo‘lishi kerak. Bu yerdan biz quyidagilarga ega bo‘lamiz $S_1''(x_0) = 0, S_N''(x_N) = 0,$ yaniy, $c_1 - d_1 h_1 = 0, c_N = 0.$

E’tibor bersak, $c_1 - d_1 h_1 = 0,$ sharti (12) tenglama bilan mos keladi.
$i = 1$ uchun, agar $c_0 = 0$ qo‘ysak, shunday qilib, kubik splaynning koeffitsiyentlarini aniqlash uchun yopiq tenglamalar tizimiga kelamiz:

$$h_i d_i = c_i - c_{i-1}, \qquad i = 1,2, \dots, N, c_0 = c_N = 0, \tag{13}$$

$$h_i c_i - \frac{h_i^2}{2} d_i = b_i - b_{i-1}, \qquad i = 1,2, \dots, N, \tag{14}$$

$$h_i b_i - \frac{h_i^2}{2} c_i + \frac{h_i^3}{6} d_i = f_i - f_{i-1}, \qquad i = 1,2, \dots, N. \tag{15}$$

Ushbu tizimning yechimi borligini ko‘rib chiqamiz. Biz $b_i, d_i,\ \ i = 1,2, \dots, N-1,$ (13) - (15) o‘zgaruvchilarni chiqarib tashlaymiz va faqat $c_i,\ \ i = 1,2, \dots, N-1,$ bo‘lgan tizimni olamiz. Buning uchun ikkita qo‘shni tenglamani ko‘rib chiqamiz (15):

$$b_i = \frac{h_i}{2} c_i - \frac{h_i^2}{6} d_i + \frac{f_i - f_{i-1}}{h_i},$$

$$b_{i-1} = \frac{h_{i-1}}{2} c_{i-1} - \frac{h_{i-1}^2}{6} d_{i-1} + \frac{f_{i-1} - f_{i-2}}{h_{i-1}}$$

va ikkinchi tenglamani birinchi tenglamadan ayiramiz.

$$b_i - b_{i-1} = \frac{1}{2}(h_i c_i - h_{i-1} c_{i-1}) - \frac{1}{6}\left(h_i^2 d_i - h_{i-1}^2 d_{i-1}\right) + \frac{f_i - f_{i-1}}{h_i} - \frac{f_{i-1} - f_{i-2}}{h_{i-1}}$$

Tenglikni chap tarafi $b_i - b_{i-1}$ o‘rniga (14) tenglamani almashtiramiz va soddalashtiramiz:

$$h_i c_i + h_{i-1} c_{i-1} - \frac{h_{i-1}^2}{3} d_{i-1} - \frac{2h_i^2}{3} d_i = 2\left(\frac{f_i - f_{i-1}}{h_i} - \frac{f_{i-1} - f_{i-2}}{h_{i-1}}\right). \tag{16}$$

Keyin, (13) tenglamani ko‘rinishini quyidagicha yozib

$$h_i^2 d_i = h_i (c_i - c_{i-1}), \qquad h_{i-1}^2 d_{i-1} = h_{i-1}(c_{i-1} - c_{i-2})$$

va (16) tenglamaga almashtirib soddalashtiramiz

$$h_{i-1}c_{i-2} + 2(h_{i-1} + h_i)c_{i-1} + h_i c_i = 6(\frac{f_i - f_{i-1}}{h_i} - \frac{f_{i-1} - f_{i-2}}{h_{i-1}}).$$

Nihoyat, c_i koeffitsiyentlarini aniqlash uchun biz tenglamalar sistemasiga egamiz.

$$h_i c_{i-1} + 2(h_i + h_{i+1})c_i + h_{i+1}c_{i+1} = 6(\frac{f_{i+1}-f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i}). \quad (17)$$

$$i = 1,2,\dots,N, c_0 = c_N = 0.$$

Diagonal tarqalganligi sababli (17) tenglamalar sistemasi yagona yechimga ega. Sistemaning matritsasi uch diogonalli bo‘lganligi uchun progonka usuli bilan yechimni topamiz. (17) tenglamani matritsada ifodalash qulay bo‘lishi uchun quyidagicha yozib olamiz.

$$A_i c_{i-1} + B_i c_i + C_i c_{i+1} = 6(\frac{f_{i+1} - f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i}).$$

$$c_0 = \omega_1 c_1 + \mu_1, c_N = \omega_2 c_{N-1} + \mu_2 \quad (18)$$

$$\begin{bmatrix} A_1 & B_1 & C_1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & A_2 & B_2 & C_2 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & A_3 & B_3 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & B_{n-2} & C_{n-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & A_{n-1} & B_{n-1} & C_{n-1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & & A_n & B_n & C_n \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_{n-3} \\ c_{n-2} \\ c_{n-1} \\ c_n \end{bmatrix} =$$

$$= \frac{6}{h_{i+1}h_i} \begin{bmatrix} h_1(f_2 - f_1) - h_2(f_1 - f_0) \\ h_2(f_3 - f_2) - h_3(f_2 - f_1) \\ h_3(f_4 - f_3) - h_4(f_3 - f_2) \\ \vdots \\ h_{n-3}(f_{n-2} - f_{n-3}) - h_{n-2}(f_{n-3} - f_{n-4}) \\ h_{n-2}(f_{n-1} - f_{n-2}) - h_{n-1}(f_{n-2} - f_{n-3}) \\ h_{n-1}(f_n - f_{n-1}) - h_n(f_{n-1} - f_{n-2}) \end{bmatrix}$$

Yuqoridagi matritsani hisoblash uchun yechimni (10) formuladan qidiramiz:

$$c_j = \alpha_{j+1_i} c_{i+1} + \beta_{j+1}, j = 0,1,\dots,N-1, \quad (19)$$

bu yerda $\alpha_{j+1_i}, \beta_{j+1}$— noma’lum bo‘lgan koeffitsiyentlar. Ularni quyidagicha aniqlaymiz:

$$c_{j-1} = \alpha_{j+1}c_j + \beta_j = \alpha_j(\alpha_{j+1}c_{j+1} + \beta_{j+1}) + \beta_j =$$

$$= \alpha_j \alpha_{j+1} c_{j+1} + (\alpha_j \beta_{j+1} + \beta_j), j = 0,1,\dots,N-1.$$

$c_{j_i}, c_{j-1}, j = 0,1,\dots,N-1$ olingan ifodalarni (17) tenglamaga almashtiramiz

$$\left[\alpha_{j+1}(h_j\alpha_j - 2(h_j + h_{j+1})) + h_{j+1}\right]c_{j+1} + \left[\beta_{j+1}(h_j\alpha_j - h_{j+1}) + h_j\beta_j + f_j\right] = 0.$$

Kvadrat qavslardagi ifodalar nolga tenglansa $\alpha_{j+1_i}, \beta_{j+1}$ koeffitsiyentlari topiladi.

$$\alpha_{j+1} = \frac{h_j}{h_{j+1} - \alpha_j h_j}, \beta_{j+1} = \frac{h_j\beta_j + f_j}{h_{j+1} - \alpha_j h_j}, j = 1,2,\ldots,N-1. \quad (20)$$

(20) tenglama α_j, β_j lar bilan ifodalanadi, ularni topish uchun dastlabki qiymatlarni belgilashimiz kerak. Bu boshlang‘ich qiymatlar $j = 0$ bo‘lganda ekvivalentlik shartidan (19) topiladi, ya’ni $c_0 = \alpha_1 c_1 + \beta_1$ ko‘rinishiga kelib, (18) tenglamani birinchisiga o‘xshash bo‘ladi.

Shunday qilib

$$\alpha_j = \omega, \beta_j = \mu_1 \quad (21)$$

(20), (21) formulalardan foydalanib $\alpha_{j+1_i}, \beta_{j+1}$ koeffitsiyentlarini topish bevosita to‘g‘ri progonka usuli deyiladi. $\alpha_{j+1_i}, \beta_{j+1}, j = 0,1,\ldots,N-1,$ koeffitsiyentlarini topish davomida, $j = N-1$ dan boshlab (17), (18) sistemaning yechimlari (20) formula bilan topiladi. Ushbu formula yordamida hisoblashlarni boshlash uchun $c_N$ma’lum bo‘lishi kerak

$$c_N = \omega_2 c_{N-1} + \mu_2, c_{N-1} = \alpha_N c_N + \beta_N$$

va u quyidagiga teng $\frac{\omega\beta_N + \mu_2}{1 - \omega_2\alpha_N}$.

Quyidagi formulalarga ko‘ra c_j topish

$$c_j = \alpha_{j+1_i} c_{i+1} + \beta_{j+1}, j = N-1, N-2, \ldots, 0,$$

$$c_N = \frac{\omega_2\beta_N + \mu_2}{1 - \omega_2\alpha_N}. \quad (22)$$

(22) ga teskari progonka usuli deyiladi. Umumiy holatda (17), (18) sistemalarni yechish algoritmi (20) - (22) formulalar bilan aniqlanadi.

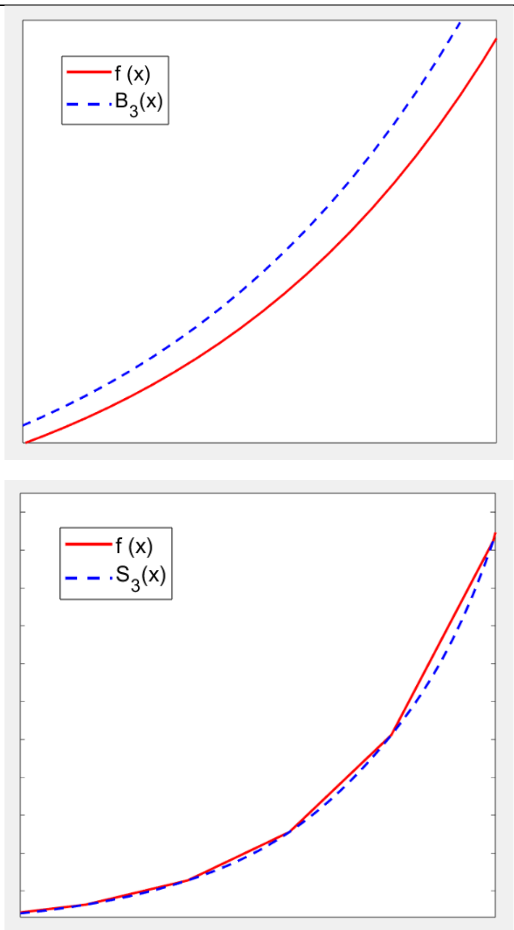
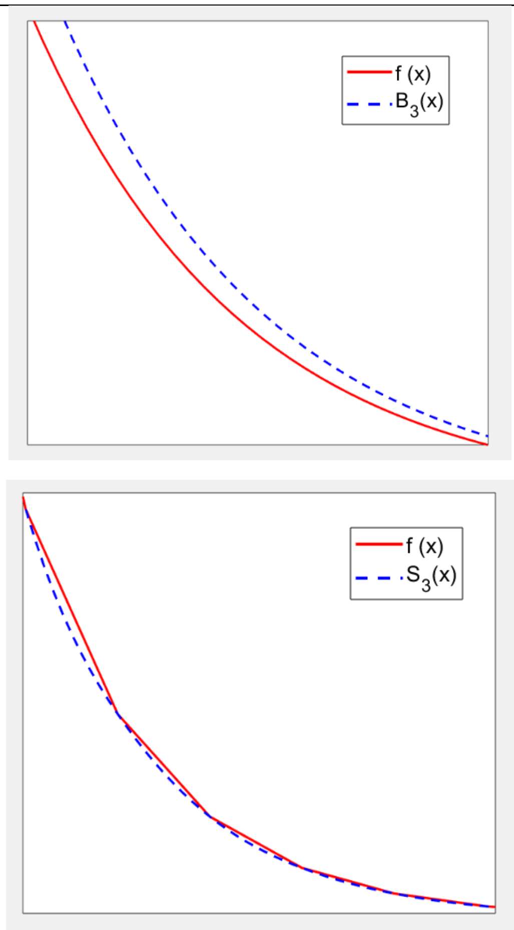
Progonka usuli orqali c_i, (i=1, … , n) koeffitsiyentlarga ega bo‘lamiz va qolgan koeffitsiyentlarni quyidagicha aniqlaymiz.

$$d_i = \frac{c_i - c_{i-1}}{h_i}, b_i = \frac{h_i}{2}c_i - \frac{h_i^2}{6}d_i + \frac{f_i - f_{i-1}}{h_i},$$

$$i = 1,2,\ldots,N.$$

Shunday qilib, $S''(a) = S''(b) = 0$ sharti bilan berilgan kubik splayn modelini qurish isbotlandi.

Yuqorida qurilishi keltirilgan lokal interpolyatsion Natural splayn-funksiya grafigi hamda kubik B -splayn funksiya grafigi bilan taqqoslandi. Ushbu ikkita lokal kubik splaynlarning yaqinlashishini berilgan funksiya $f(x)$ bilan solishtiramiz:

$f(x) = 2^x$ bo‘lganda	$f(x) = 2^{-x}$ bo‘lganda
	
3-rasm. $f(x) = 2^x$ funksiyani ikkita kubik splayn bilan yaqinlashtirish natijalari	**4-rasm. $f(x) = 2^{-x}$ funksiyani ikkita kubik splayn bilan yaqinlashtirish natijalari**

1, 2 -misollardan ko‘rinadiki, $S_3(x)$ splayn funksiya grafigi $f(x)$ funksiya grafigini $B_3(x)$ splayn funksiya grafigiga qaraganda yaxshiroq yaqinlashtiradi. Bu esa signallarga raqamli ishlov berishda Natural splaynning aniqlik ko‘rsatkichi yuqoriligini va qurilish jihatidan soddaligini hisobga olib hisoblash jarayonlari uchun qo‘l kelishini ko‘rishimiz mumkin.

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